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The Systems Engineering Tool Box

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"Give us the tools and we will finish the job"
Winston Churchill

The Analytic Hierarchy Process (AHP)

What is it and what does it do?

The Analytic Hierarchy Process (AHP) is a tool that provides an effective structure for group decision-making [1]. It can be used to:

- Select between a number of options
- Prioritise requirements/criteria.

The Analytic Hierarchy Process helps the group assign numerical values to subjective judgments about a number of criteria and consequently combining the judgments into a single scale for decision-making.

Why do it?

Decisions often concern a number of criteria whose selection is at the bidding of the decision-makers. These various criteria are likely to be measured on different scales, such as weight and height, or are intangible, as no scales currently exist. Ideally, we require a way of combining these criteria in a meaningful way.

The Analytic Hierarchy Process provides a structured and disciplined approach to taking these multiple criteria into account when reaching a decision providing a result on a single scale. It allows us to make a judgement. Judgements are the ability to make considered decisions or come to sensible conclusions that can take several forms:

a. The mental ability to perceive and distinguish relationships*. For example; tiredness may affect a driver's judgment of speed*
b. The ability to form an opinion by distinguishing and evaluating*. For example; he felt that this gig was better than the band's last one at this venue because the sound quality was superior*
c. The capacity to assess situations or circumstances and draw sound conclusions. *For example; given the time of day, and the distance we have walked, I propose a taxi home would be sensible*

Judgments, however, come in two basic forms: absolute and relative.
An *absolute judgment* is where it is possible to identify the magnitude of something – for instance, the loudness of a sound or the size of a space. Such judgments are usually in terms of standards in memory about similar "things". For example, this

restaurant is twice as big as the one in Derby, this car is 25% more expensive than that one.

A *relative judgment* is the identification of some relation between two “things” that are both present to the observer-judge. For example, that shirt is “bluer” than that one, you are taller than your friend.

Decision making tools like Pairwise Comparisons and Pugh Matrices rely on our ability to make relative judgments. Actually, we are rather good at it. Given two choices and a decision criterion we can usually pick the winner. Decision tools based on using relative judgments unfortunately suffer from inconsistency, particularly where the decision involves several criteria – which of course is the everyday reality we all face. Inconsistency can be described as:

- A is twice as good as B
- B is twice as good as C
- C is twice as good as A.

Unwittingly, when making decisions that involve several criteria we are frequently inconsistent despite our good intentions.

The beauty of the AHP is that it gets us to use absolute judgments, which in turn enables us to compute how consistent we have been. Consistency is a necessary but not sufficient condition in decision making and therefore presents a superior approach over other decision making approaches.

Where and when to use it?

Fundamentally the AHP can be used whenever there is the need to decide amongst a number of alternatives or prioritise a number of factors or requirements.

Who does it?

An individual or team can use the AHP. It is important to emphasise, however, that the quality of the outcome is dependent upon the experience of team or individual.

How to do it?

The AHP is a four-step process as shown in Figure 1.

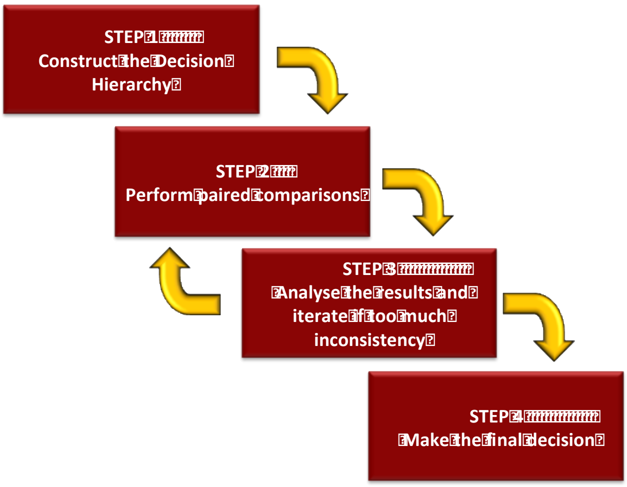

Figure 1: The 4-step Analytic Hierarchy Process

The following will describe the 4-step process shown in Figure 1. This description is deliberately limited to a non-mathematical treatment, it also assumes that you the reader has access to relevant software for performing the necessary calculations. The AHP has a mathematical foundation and can be explained from a mathematical viewpoint, however, results and decisions can be made without and understanding of the underlying mathematics. Appendix A provides a semi-mathematical description of the AHP and Appendix B provides a description of how approximate the AHP calculations in Excel.

Step 1: Construct The Decision Hierarchy

The Analytic Hierarchy Process requires us to decompose the decision problem into a hierarchy of more easily comprehended sub-problems, each of which can be analysed independently. The elements of the hierarchy can relate to any aspect of the decision problem whether tangible or intangible, carefully measured or roughly estimated, well, or poorly, understood - indeed anything at all that applies to the decision at hand.

The Decision Hierarchy comprises the “goal” at the top of the hierarchy. The second level of the hierarchy consist a number of primary criteria at a similar level of importance. The next level contains, if appropriate, the sub-criteria for each criterion in the level above. Again these sub-criteria should be at a similar level of importance to each other. This 3-level hierarchy is shown in Figure 2.

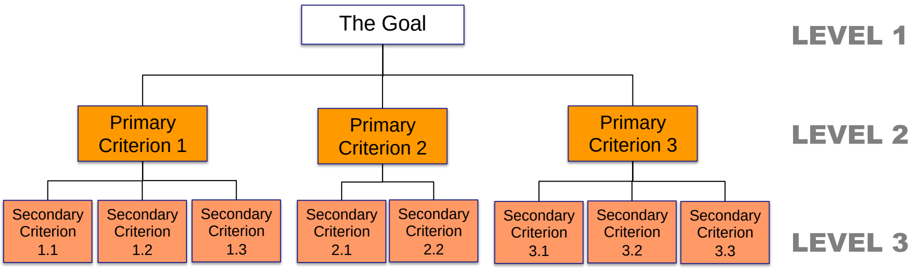

Figure 2: A 3-level AHP Decision Hierarchy

For example, let us suppose that we are trying to prioritise a set of user/customer requirements for a washing machine. In this case the requirements have been gathered via a small group of Users using an Affinity Diagram[1] and Tree Diagram[2]. The outcome is shown in Figure 3.

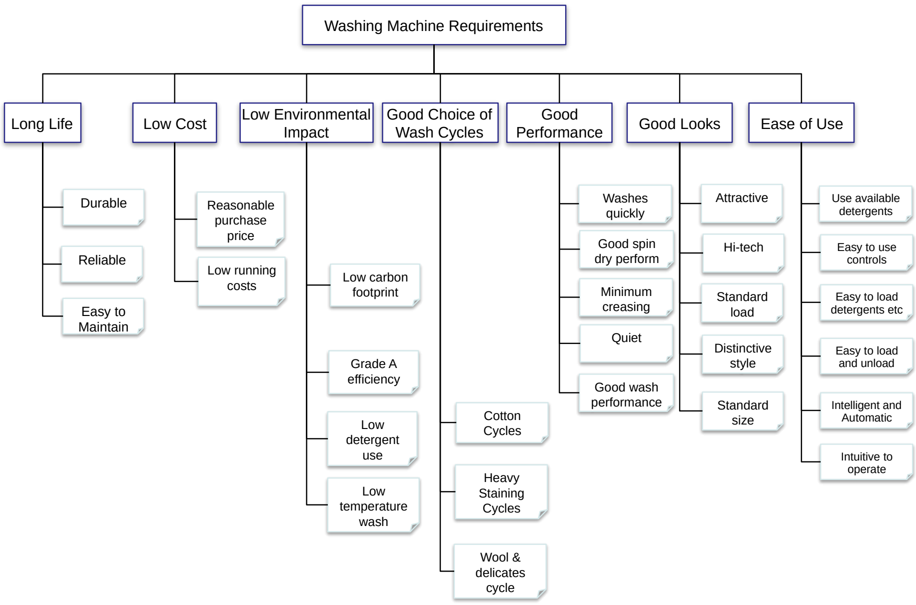

Figure 3: A Tree Diagram of Requirements for a Washing Machine that can be used as an AHP Decision Hierarchy

Notice with the example in Figure 3 some of the criteria are measurable (Long Life, low cost etc.) but others are less tangible (good looks, ease of use etc.) It is this ability to "mix" quantitative and qualitative criteria that makes the AHP very powerful.

[1] An Affinity Diagram is a simple Systems tool that allows a team or group to: generate ideas about a situation or problem and organize these ideas into meaningful groups [2].

[2] A Tree Diagram is also a simple tool that allows the team or group to represent ideas in a simple hierarchy [2].

Step 2: Perform Paired Comparisons

To perform the AHP the first level of criteria (requirements) at are systematically subject to pairwise comparisons using an NxN matrix – where N is the number of criterion being prioritised. In the case of the washing requirements we need to establish a 7x7 Comparison Matrix as shown in Figure 4.

	Long life	Low Cost	Low Env. Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life							
Low Cost							
Low Env. Impact							
Choice of Cycles							
Good wash							
Looks good							
Ease of Use							

Figure 4: The 7x7 Comparison Matrix for the washing machine requirements

This 7x7 matrix is used to capture and record the pairwise comparisons between the requirements according to the scale given in Table 1.

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two factors contribute equally to the objective
3	Somewhat more important	Experience and judgment slightly favour one factor over the other
5	Much more important	Experience and judgment strongly favor factor one over the other
7	Very much more important	A factor is favoured very strongly over the other. Evidence exists for its dominance
9	Extremely more important	The evidence favoring one factors over the other is of the highest possible validity
2,4,6,8	Intermediate Importance	For compromise between the above values

Table 1: Standard AHP scale for making paired comparisons

Paired comparisons are then performed on the criteria (requirements). In performing the paired comparisons **order is important – it is always the rows versus the columns.**

When performing the paired comparisons the phrasing of the question is also VERY important. If we were comparing the criteria/requirements of "*long life*" and "*low cost*" we would ask:

> How much more important to the customer is improved "long life" than improved "low cost"?

Note that the Comparison Matrix shown in figure 3 requires us to compare "long life" with "long life", "low cost" with "low cost" etc. Hence the leading diagonal of the Comparison Matrix will from Table 1 contain 1s as shown in Figure 5 because "long life" is equally important as "long life"

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life	1						
Low Cost		1					
Low Env Impact			1				
Choice of Cycles				1			
Good wash					1		
Looks good						1	
Ease of Use							1

Figure 5: The identity aspect of the Comparison Matrix

Because the lower triangle of the Comparison Matrix is the reciprocal of the upper triangle, we only need to complete one or the other. Typically, because the comparison is rows versus columns it is logical to complete the upper triangle. As noted above it is very important when conducting the pairwise comparison to be clear on the question. For example:

> How much more important to the customer is improved "long life" than improved "low cost"?

The Analytic Hierarchy Process requires us to make a choice from 18 the possibilities given in Table 1. Let us suppose the decision made is:

> *Somewhat more important*

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From table 1 we can complete two cells of the matrix as shown in Figure 6.

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life	1	3					
Low Cost	0.333	1					
Low Env Impact			1				
Choice of Cycles				1			
Good wash					1		
Looks good						1	
Ease of Use							1

Figure 6: Reciprocal Nature of the Comparison Matrix

The 3 in the second cell of the first row becomes 0.333 in first cell of the second row.

When doing the comparison it is possible that the row criterion is less important than the column criterion. In such cases we use Table 1 to determine the reverse – i.e. how much more important the column criterion is than the row criterion but put the reciprocal from Table 1 in the cell. For example, consider the question:

How much more important to the customer is improved “low environmental impact” than improved “choice of cycle”?

In this case the team decide that improved “choice of cycle” is *much more important* than improved “low environmental impact”. Hence, the cell will contain the reciprocal of 3 = 0.333 to reflect this judgement.

Figure 7 shows the completed Comparison Matrix. Note that only the upper triangle was completed – the lower being the reciprocal.

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life	1	3	3	3	3	0.143	0.2
Low Cost	0.333	1	3	3	1	0.2	0.333
Low Env Impact	0.333	0.333	1	0.333	1	0.143	0.2
Choice of Cycles	0.333	0.333	3	1	1	0.2	0.333
Good wash	0.333	1	1	1	1	0.2	0.333
Looks good	7	5	7	5	5	1	3
Ease of Use	5	3	5	3	3	0.333	1

Figure 7: The Completed Comparison Matrix for the Washing Machine Requirements

Perhaps more important was the questions that were asked to elicit the responses. It is critical that the team/group conducting the AHP do NOT interpret the question as:

is requirement A more important than Requirement B?

To this end it is often desirable (if not essential) the AHP session is facilitated. Moreover, that the questions are constructed and written beforehand, and even issued to the team/group members. A good practice is to construct a table like that shown in Figure 8 to capture the questions and ensure the correct question is asked.

Qu. No.	Question
1	How much more important to the customer is improved "Long Life" than improved "Low Cost"?
2	How much more important to the customer is improved "Long Life" than improved "Low Environmental Impact"?
3	How much more important to the customer is improved "Long Life" than improved "Choice of Cycles"?
4	How much more important to the customer is improved "Long Life" than improved "Good Wash"?
5	How much more important to the customer is improved "Long Life" than improved "Good Looks"?
6	How much more important to the customer is improved "Long Life" than improved "Ease of use"?
7	How much more important to the customer is improved "Low Cost" than improved "Low Environmental Impact"?
8	How much more important to the customer is improved "Low Cost" than improved "Choice of Cycles"?
. . .	
21	How much more important to the customer is improved "Looks Good" than improved "Ease of Use"?

Figure 8: The table of questions for the AHP of a set of Washing Machine Requirements

Having completed the Comparison Matrix it is now a question of Mathematics. Ideally, a software package is needed to perform the necessary calculations. Alternately, Appendix B presents an approximate route using Excel.

Using the Qualica software package the following results were obtained from the data given in Figure 7.

Criteria	Importance
Long Life	12%
Low Cost	7%
Low Env Impact	4%
Choice of Cycles	6%
Good wash	6%
Looks good	42%
Ease of Use	23%

This information is also supported by a calculated *Consistency Ratio* of 0.08. This particular figure is important since it is a measure of how consistent the team has been in making the judgements in the Comparison Matrix (figure 6). In this instance because the *Consistency Ratio* < 0.1. If the *Consistency Ratio* is above 0.1 it is possible to conclude that the team was not consistent in making its judgements. If this does occur, the Comparison Matrix should be investigated to identify the source of the inconsistency.

What Goes Wrong: The limitations of the Analytic Hierarchy Process

Decision criteria. The "quality" of the decision using the Analytic Hierarchy Process is fundamentally related to the "quality" of the selection criteria. This has three aspects:

Level of hierarchical importance: When constructing the Decision Hierarchy it is important to ensure that the criteria sit in levels that have the same importance. For example having "easy to load" at the same level as "easy to use" will require us to make a comparative judgement between the two. Yet one is a sub-criterion of the other, and while a judgement can be made it will be heavily biased potentially leading to the wrong decision.

Too many criteria in one level: The mathematics of the AHP are such that if we have more than seven criteria in a Comparison Matrix we are not sufficiently sensitive to make accurate changes in judgment on so criteria simultaneously. In simple terms we should ensure that level 2 of the Decision Hierarchy should contain 7 or fewer criteria. In a similar vein, each criterion in level 2 it should comprise 7 or fewer level 3 criteria. You can have as many levels in the Decision Hierarchy as you like.

Inadequately defined criteria: Poorly defined criteria can result in unstated multiple interpretations. It is important to put the time and effort into determining the criteria and to consider some form of validation.

Inadequately posed questions: The questioning in the AHP is very important. It is possible through poor question construction and order of comparison to obtain consistent judgements that are wrong. It is important to follow the guidance given above and in particular the recommendation to write all the questions out fully before the paired comparisons session.

Wrong expertise and insufficient experience in teams. Like a great many Systems Engineering tools, the Analytic Hierarchy Process is really only a vehicle to help extract the knowledge and experience from the team. The wrong team can still follow the process and arrive at a result – but the result may not robust.

Success Criteria

The following list represents a set of criteria that have been found to be useful when using the Analytic Hierarchy Process.

- Team size between 4 and 8.
- Team constitution has expertise and experience in the system of interest but can (and perhaps should) include members with limited experience and expertise.
- Use an experience independent facilitator.
- Plan for 2-3 hours effort.
- Define clearly what we are trying to do
- Have validated and weighted decision criteria available (the “customer requirements” from Quality Function Deployment 1 for example).
- Spend time to check-out the team’s understanding of the criteria and if necessary clearly define (and document) the agreed understanding (very useful in subsequent design reviews).
- Have all the pairwise comparison questions formed and written down before the session.
- Document any debate.

Appendix A: A Semi-Theoretical Explanation of AHP

The AHP is actually a fascinating piece of work that ties together several aspects pertinent to Systems Engineering. These are:

a) Managing complexity: here George Miller's 7 ± 2 is often used to manage complexity
b) Managing risk: Systems Engineering can be considered to be a set of decisions that have various degrees of uncertainty that is related to the time line of a project/programme

The AHP provides a theoretical underpinning for both aspects. This short note aims to bridge the gaps between pure mathematical descriptions of AHP and the "just do this" process based descriptions.

Judgements

To start this semi theoretical explanation of AHP I want to talk about human judgements. Our ability to make, or perhaps not make, a judgement is central to the AHP.

A judgement is the ability to make considered decisions or come to sensible conclusions that can take several forms:

a. The mental ability to perceive and distinguish relationships*. For example; tiredness may affect a driver's judgment of speed.*

b. The ability to form an opinion by distinguishing and evaluating*. For example; he felt that this gig was better than the band's last one at this venue because the sound quality was superior.*

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Judgments, however, come in two basic forms: absolute and relative:

An *absolute judgment* is where it is possible to identify the magnitude of something – for instance, the brightness of a light, the loudness of a tone, or the curvature of a line. Such judgments are usually in terms of standards in memory about similar "somethings". For example, This restaurant is twice as big as the one in Manchester, this car is 25% more expensive than that one.

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Unwittingly, when making decisions that involve several criteria we are frequently inconsistent despite our good intentions.

The beauty of the AHP is that it gets us to use absolute judgments, which in turn enables us to compute how consistent we have been. Consistency is a necessary (but not sufficient) condition in decision making and therefore presents a superior approach over other decision making approaches.

To explain the AHP I will use an example for which we know the correct answer. Imagine there are five cylinders with different heights as shown in figure A1.

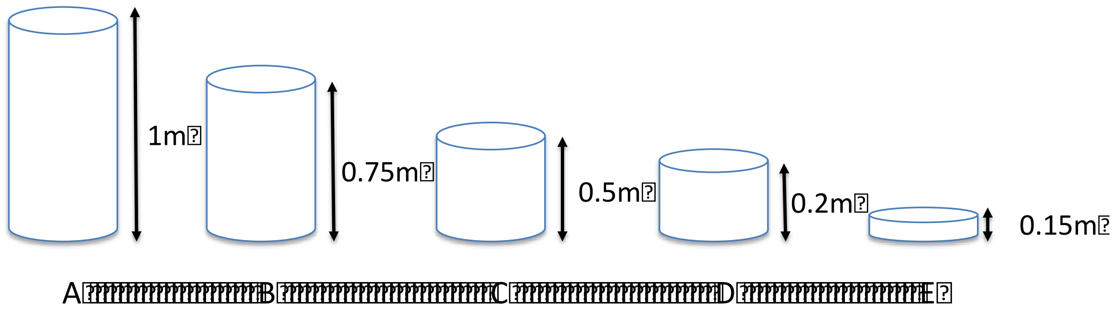

Figure A1: Five cylinders of different but known height

If we were making relative judgments, we would be able to say that cylinder A is taller than cylinder C and so on. However, in this case (because we the actual heights) we can make absolute judgments and make statements like

Cylinder A is 1.333 times taller than cylinder B
Cylinder A is 2 times taller than cylinder C

Given the five cylinders it is possible to construct a 5x5 matrix that contains all the absolute judgments between the various cylinder heights

$$\begin{bmatrix} h_A/h_A & h_A/h_B & h_A/h_C & h_A/h_D & h_A/h_E \\ h_B/h_A & h_B/h_B & h_B/h_C & h_B/h_D & h_B/h_E \\ h_C/h_A & h_C/h_B & h_C/h_C & h_C/h_D & h_C/h_E \\ h_D/h_A & h_D/h_B & h_D/h_C & h_D/h_D & h_D/h_E \\ h_E/h_A & h_E/h_B & h_E/h_C & h_E/h_D & h_E/h_E \end{bmatrix}$$

For our cylinder example this matrix would be

$$\begin{bmatrix} 1 & 1.333 & 2 & 5 & 6.666 \\ 0.75 & 1 & 1.5 & 3.75 & 5 \\ 0.5 & 0.666 & 1 & 2.5 & 3.333 \\ 0.2 & 0.266 & 0.4 & 1 & 1.333 \\ 0.15 & 0.2 & 0.3 & 0.75 & 1 \end{bmatrix}$$

This matrix has a special structure since $h_i/h_j = {}^1/_{h_j/h_i}$. Also $h_i/h_i = 1$ giving

$$\begin{bmatrix} 1 & h_A/h_B & h_A/h_C & h_A/h_D & h_A/h_E \\ {}^1/_{h_A/h_B} & 1 & h_B/h_C & h_B/h_D & h_B/h_E \\ {}^1/_{h_A/h_C} & {}^1/_{h_B/h_C} & 1 & h_C/h_D & h_C/h_E \\ {}^1/_{h_A/h_D} & {}^1/_{h_B/h_D} & {}^1/_{h_C/h_D} & 1 & h_D/h_E \\ {}^1/_{h_A/h_E} & {}^1/_{h_B/h_E} & {}^1/_{h_C/h_E} & {}^1/_{h_D/h_E} & 1 \end{bmatrix}$$

This reciprocal matrix has an interesting eigenstructure. It is assumed at this point that you the reader has encountered linear algebra and is familiar with the basics of eigenvalues and eigenvectors – if not either stop at this point and accept that the AHP works (provided you follow the rules) or get into basic linear algebra.

If the eigenvalues of the above are calculated they will be (5, 0, 0, 0, 0) irrespective of the values of h_i/h_j. In fact if we had *n* cylinders and computed the equivalent matrix the eigenvalues would be (*n*, 0, 0, … ,0). The non-zero eigenvalue is called λ_{max}. Perhaps more interesting is that the eigenvector associated with the non-zero eigenvalue (i.e. in this example the 5) will be:

$$\begin{bmatrix} 1 \\ h_A/h_B \\ h_A/h_C \\ h_A/h_D \\ h_A/h_E \end{bmatrix}$$

The implication of this vector is quite profound because it is the absolute judgements about the heights of the cylinders. In this example of course we knew these, but when applying the AHP to real world decision making the issue is that they absolute judgements are not known – but we can ask a group of experts to give their considered absolute judgements. There is no doubt that this group will be making judgements as opposed to measurements (like in our cylinder example) and therefore there are issues of uncertainty. This uncertainty has several dimensions but one that can be calculated is the consistency. The concept of consistency was introduced early but we can now define it mathematically as

$$h_i/h_k = (h_i/h_j)(h_j/h_k)$$

This equation is not obviously intuitive (well I don't think so) but it mathematically states the example of consistency given earlier.

Let us suppose that the cylinders actually exist and a group is asked to make judgements about the heights just by simple comparison. In posing the problem we would ask the group very specific questions around which they would make their judgements. Moreover the questions only need to be asked for the upper (or lower) triangle because the other triangle will be the reciprocals. The questions would be – and their answers!!

Question	Answer
How much taller is cylinder A than cylinder B?	1.25
How much taller is cylinder A than cylinder C?	2
How much taller is cylinder A than cylinder D?	5
How much taller is cylinder A than cylinder E?	7
How much taller is cylinder B than cylinder C?	1.5
How much taller is cylinder B than cylinder D?	4
How much taller is cylinder B than cylinder E?	5
How much taller is cylinder C than cylinder D?	2.5
How much taller is cylinder C than cylinder E?	3
How much taller is cylinder D than cylinder E?	1.4

Which means we can formulate the reciprocal matrix

$$\begin{bmatrix} 1 & 1.25 & 2 & 5 & 7 \\ 0.8 & 1 & 1.5 & 4 & 5 \\ 0.5 & 0.666 & 1 & 2.5 & 3 \\ 0.2 & 0.25 & 0.4 & 1 & 1.4 \\ 0.143 & 0.2 & 0.333 & 0.714 & 1 \end{bmatrix}$$

This matrix is slightly different from the earlier one and therefore will have a different eignestructure.

For a consistent matrix, the non-zero eigenvalue = n (5 in our case). For matrices involving human judgment, the condition $h_i/h_k = (h_i/h_j)(h_j/h_k)$ does not hold as human judgments are inconsistent to a greater or lesser degree. In such a case we find that the actual eigenvalue $\lambda_{max} \geq n$. The difference, if any, between λ_{max} and *n* is an indication of the inconsistency of the judgments. If $\lambda_{max} = n$ then the judgements have turned out to be consistent.

To make the AHP easy to use a ***Consistency Index*** can be calculated from *(λmax-n)/(n-1)*. This can be assessed against judgments made completely at random! The originator of the AHP, Saaty, calculated large samples of random matrices of increasing order and the Consistency Indices of those matrices. Table 1 below shows the results that Saaty obtained from random simulations. What he did was to choose the entries for different size of matrix (up to 15x15) and randomly assign above main diagonal {1/9, 1/8,...,1, 2,...,8, 9}. The entries below the diagonal were calculated by taking reciprocals. The main diagonal was completed as 1.0s. These matrices were used to compute the consistency index. Amazingly Saaty did this 50,000 times and used the average which he called the *random index*. The results are given in Table A1 below:

Order	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Random Index	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49	1.52	1.54	1.56	1.58	1.59
First order differences		0	0.52	0.37	0.22	0.14	0.1	0.05	0.05	0.04	0.03	0.02	0.02	0.02	0.01

Table A1: The AHP Random Index Values

A true ***Consistency Ratio*** is calculated by dividing the ***Consistency Index*** for the set of judgments by the value for the corresponding ***Random Index***. That is:

$$CR = {}^{CI}/_{RI}$$

Saaty suggests that if that the Consistency Ratio exceeds 0.1 the set of judgments may be too inconsistent to be reliable. This cutoff of 0.1 come from the concept of order of magnitude and is essential in any mathematical consideration of changes in measurement. Suppose we have a numerical value say between 1 and 10 for some measurement and we want to know whether change in this value is significant or not, the following reasoning is usually applied:

- A change of a whole integer value is critical because it changes the magnitude and identity of the original number significantly.
- If the change in value is of the order of a percent or less, it would be so small (by two orders of magnitude) and would be considered negligible.

In attempting to make consistent judgments, changes that are too large can cause dramatic change in our understanding, and values that are too small cause no change in our understanding. In between we are left with only values of one order of magnitude smaller that we can deal with incrementally to change our understanding. It follows that our allowable consistency ratio should be not more than about .10. The requirement of 10% cannot be made smaller such as 1% or .1% without trivializing the impact of inconsistency.

In practice, Consistency Ratios of more than 0.1 sometimes have to be accepted. If the Consistency Ratio equals 0 then that means that the judgments are perfectly consistent. Indeed if we return to the 5 Cylinder problem then

$$CR = {}^{(\lambda max\text{-} n)/(n\text{-}1)}/_{1.11} = {}^{(5-5)/4}/_{1.11} = 0$$

Which is what we would expect because we know the exact measurements of the cylinders.

Another interesting features comes from the first order differences of the Random Indices given in Table A1. If these are plotted against the order number the following figure 2 results.

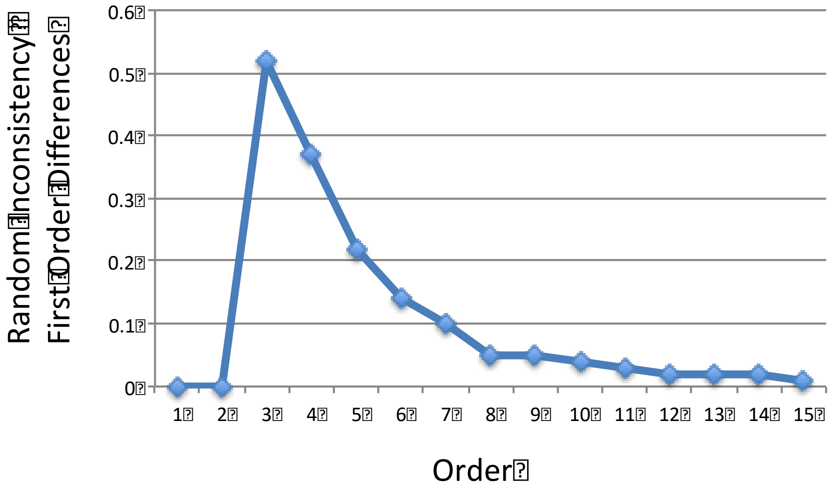

Figure A2: Random Inconsistency first order differences

Figure A2 is a plot of these differences and shows the importance of the number seven as a cutoff point beyond which the differences are less than 0.10 where we are not sufficiently sensitive to make accurate changes in judgment on several elements simultaneously.

Appendix B: An Approximate Method of Calculation for AHP using Excel

The mathematics used The Analytic Hierarchy Process requires is reasonably involved and not easily performed by hand or by Excel. There are software packages available that employ these precise approaches, however it is possible to employ a manual, approximate method since it is very easy to do in Excel and comprises two basic steps:

- Calculation of the importance weighting
- Calculation of the consistency ratio

Step 1 Calculate the Importance Weighting

Step 1.1: Calculate the sum of columns of Comparison Matrix

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life	1	3	3	3	3	0.143	0.2
Low cost	0.333	1	3	3	1	0.2	0.333
Low Env Impact	0.333	0.333	1	0.333	1	0.143	0.2
Choice of Cycles	0.333	0.333	3	1	1	0.2	0.333
Good wash	0.333	1	1	1	1	0.2	0.333
Looks good	7	5	7	5	5	1	3
Ease of Use	5	3	5	3	3	0.333	1
Column Sum	14.332	13.666	23	16.333	15	2.219	5.399

Step 1.2: Normalise the entries in each column by the sum of the column concerned:

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life	0.06977393	0.2195229	0.13043478	0.18367722	0.2	0.06444344	0.0370439
Low cost	0.02323472	0.0731743	0.13043478	0.18367722	0.06666667	0.09013069	0.06167809
Low Env Impact	0.02323472	0.02436704	0.04347826	0.02038817	0.06666667	0.06444344	0.0370439
Choice of Cycles	0.02323472	0.02436704	0.13043478	0.06122574	0.06666667	0.09013069	0.06167809
Good wash	0.02323472	0.0731743	0.04347826	0.06122574	0.06666667	0.09013069	0.06167809
Looks good	0.48841753	0.36587151	0.30434783	0.3061287	0.33333333	0.45065345	0.55565846
Ease of Use	0.34886966	0.2195229	0.2173913	0.18367722	0.2	0.1500676	0.18521949

Step 1.3: Calculate the arithmetic average of the rows to produce an estimate of the importance weighting:

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use	Importance
Long Life	0.06977393	0.2195229	0.13043478	0.18367722	0.2	0.06444344	0.0370439	0.129
Low cost	0.02323472	0.0731743	0.13043478	0.18367722	0.06666667	0.09013069	0.06167809	0.090
Low Env Impact	0.02323472	0.02436704	0.04347826	0.02038817	0.06666667	0.06444344	0.0370439	0.040
Choice of Cycles	0.02323472	0.02436704	0.13043478	0.06122574	0.06666667	0.09013069	0.06167809	0.065
Good wash	0.02323472	0.0731743	0.04347826	0.06122574	0.06666667	0.09013069	0.06167809	0.060
Looks good	0.48841753	0.36587151	0.30434783	0.3061287	0.33333333	0.45065345	0.55565846	0.401
Ease of Use	0.34886966	0.2195229	0.2173913	0.18367722	0.2	0.1500676	0.18521949	0.215

Note the exact method gives

Estimated Importance	Actual Importance
0.13	0.12
0.09	0.07
0.04	0.04
0.07	0.06
0.06	0.06
0.40	0.42
0.21	0.23

Step 1 Calculate the Consistency Ratio

Step 1 Calculate the Consistency Ratio

Step 2.1: Multiply each column of the paired Comparison Matrix by its corresponding importance weighting. (Note: not matrix multiplication!)

1
0.333
0.333
0.333
0.333
7
5

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life	0.12927088	0.26956991	0.11983809	0.19617331	0.17982363	0.05729011	0.0429928
Low cost	0.0430472	0.08985664	0.11983809	0.19617331	0.05994121	0.08012602	0.07158302
Low Env Impact	0.0430472	0.02992226	0.03994603	0.02177524	0.05994121	0.05729011	0.0429928
Choice of Cycles	0.0430472	0.02992226	0.11983809	0.0653911	0.05994121	0.08012602	0.07158302
Good wash	0.0430472	0.08985664	0.03994603	0.0653911	0.05994121	0.08012602	0.07158302
Looks good	0.90489618	0.44928319	0.2796222	0.32695552	0.29970605	0.40063011	1.20189034
Ease of Use	0.64635441	0.26956991	0.19973014	0.19617331	0.17982363	0.13340983	0.21496402

Step 2.2: Sum the rows of this matrix

	Long life	Low Cost	Low Env Impact	Choice of Cycles	Good Wash	Looks Good	Ease of Use
Long Life	0.12927088	0.26956991	0.11983809	0.19617331	0.17982363	0.05729011	0.0429928
Low cost	0.0430472	0.08985664	0.11983809	0.19617331	0.05994121	0.08012602	0.07158302
Low Env Impact	0.0430472	0.02992226	0.03994603	0.02177524	0.05994121	0.05729011	0.0429928
Choice of Cycles	0.0430472	0.02992226	0.11983809	0.0653911	0.05994121	0.08012602	0.07158302
Good wash	0.0430472	0.08985664	0.03994603	0.0653911	0.05994121	0.08012602	0.07158302
Looks good	0.90489618	0.44928319	0.2796222	0.32695552	0.29970605	0.40063011	1.20189034
Ease of Use	0.64635441	0.26956991	0.19973014	0.19617331	0.17982363	0.13340983	0.21496402

Step 2.3: Divide this Row Sum column by the Importance Weightings, element for element, to create a new column

Row Sum		Importance		
0.994958734		0.129		7.69669639
0.660565492		0.090		7.351326589
0.294914851	÷	0.040	=	7.382832812
0.469848906		0.065		7.185211401
0.449891227		0.060		7.50554137
3.862983587		0.401		9.642269665
1.840025261		0.215		8.559689306

Step 2.4: Take the average of this column: λ_{max} = 7.90

Step 2.5: Compute the Consistency Index CI = (λ_{max} – N)/(N – 1) = (7.9 -7)/ 6 – 1) = 0.150
Step 2.6: Look up CI_{rand}, the consistency index for a random matrix of size N from the table:

size of matrix	1	2	3	4	5	6	7	8	9	10
random consistency index	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

In this case N = 7, hence CI_{rand} = 1.35

Step 2.7: Compute the **Consistency Ratio** as CI/CL_{rand}

CR = 0.15/13.5 = 0.11

The actual CR, computed using the precise method, was 0.08. Note that we usually test against 10%. If CR > 10% we would suspect inconsistency. In this case the actual CR is below (8%) while the estimated CR above (11%). We need to be intelligent in using these values.