The Systems Engineering Tool Box

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"Give us the tools and we will finish the job"
Winston Churchill

N^2 Analysis (N^2 or N2) – Alias Design Structure Matrix (DSM)

What is it and what does it do?

N^2 Analysis is a tool that uses a nxn matrix to record the interconnections between elements of a system. It has a number of potential uses:

- In system design to assess the degree of binding and coupling in a system and thereby determine candidate architectures based on the natural structure of the system.
- In systems design to record, and thence aid the management of, the interfaces in a system.
- In systems analysis to identify and document the interconnectivity in a system to help understand observed behaviour and to provide guidance for improvement.

Why do it?

The behaviour of system is profoundly driven by the interconnectivity of the elements of that system. Understanding this is of paramount importance, particularly in system design. N^2 Analysis can help identify the natural interconnectivity that exists in all systems and subsequently exploit this in determining the most appropriate architectural design or to analyse and understand the behaviour of an existing system. Figure 1 shows a typical N^2 matrix that comprises six elements A to F. An element can be a system function or a physical entity. The elements are record on the leading diagonal of the matrix.

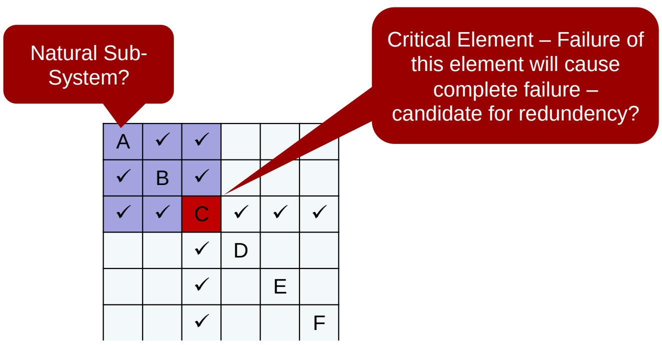

Figure 1: A Typical N^2 Matrix

The “ticks” in the cells of the matrix indicate the presence of an interconnection between system elements. By definition, the outputs are in row and the inputs are in columns. Hence the A element row has two ticks signifying the element A is “outputting” to elements B and C. The ABC block of elements is shown as being tightly bound and from a system design viewpoint is a candidate sub-system.

An N^2 Matrix can also indicate the presence of “critical elements”. In Figure 1, element C “outputs” to every other element and receives an input from every other element. If element C was to fail, then the whole system could fail. Element C is therefore a “weak link” and where redundancy might be employed.

Any convenient symbol can be used to indicate interconnections. A common approach is to use the number 1. Some N^2 Analysts use the numbers 0 to 9 to indicate not only the presence, but also the strength of the interconnection. Other symbols can be used to indicate different types interconnection. This is particularly appropriate when using N^2 to record the actual interfaces in a system. Here different symbols can be used to indicate mechanical, electrical, information or control interfaces. There are no recognised standard conventions for the different types of interfaces.

N^2 Analysis can be applied top-down as shown in Figure 2.

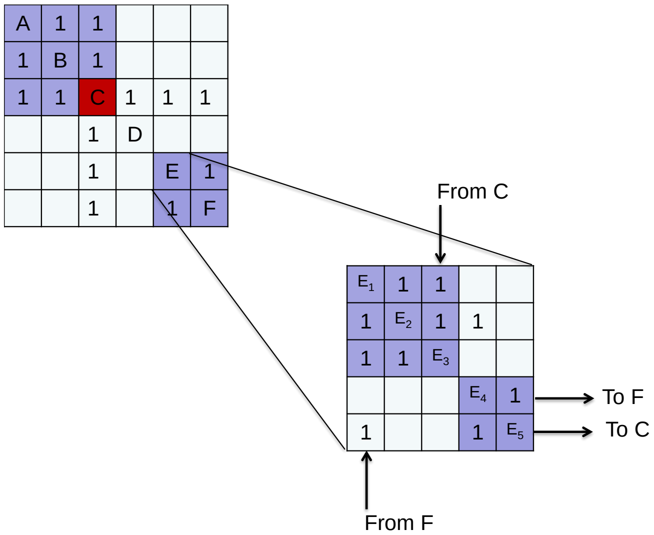

Figure 2: The top-down application of N^2 Analysis

In Figure 2, element E has been is shown to have its own N^2 Matrix that shows the interconnectivity of the sub-elements of E. The arrows around the periphery of the E sub-element matrix indicate the inputs and outputs captured on the high-level diagram.

Where and when to use it?

N^2 Analysis has several uses in Systems Engineering:

- During system design to analyse and uncover the natural structure hidden inside system problems. In combination with functional modelling tools, like Function Flow Diagrams, N^2 Matrices are a very powerful, yet simple system architecting tool to identify potential sub-systems and critical system elements.
- To analyse potential or existing systems to understand behaviour and identify system weaknesses.
- To document the system interfaces in a simple and readable format.
- To help in the management of interfaces throughout the life of a system. They can enhance Interface Definition or Control Documents by providing a simple clear overall picture.

Who does it?

An individual or team can perform N^2 Analysis. When developing higher-level system architectures there is benefit in involving a team since the analysis may well generate a number of candidate architectures that will require evaluation and ultimately down-selection. At lower-levels, an individual can readily perform the analysis.

How to do it?

N^2 Analysis can be used in several situations. In the following, these will be considered separately although there is overlap in approach and outcomes. There are however some basic features that are common.

Basic Features and Rules Governing the Construction of N^2 Matrices

Figure 3 shows the key features and rules of an N^2 matrix. The rules are:

- System Elements or Functions are on the leading diagonal.
- System Elements or Functions have inputs and outputs.
- Outputs are contained in rows; inputs are contained in columns.
- External Inputs and Outputs are shown entering and leaving the N^2 matrix in-line with their respective source or destination function.

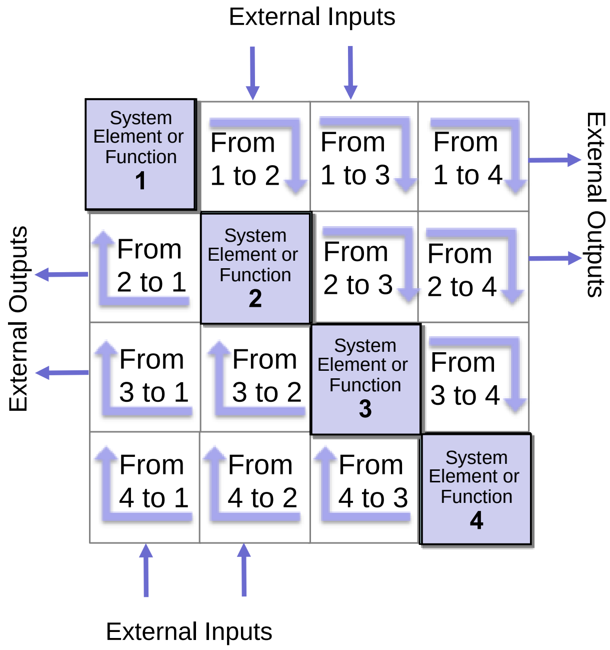

Figure 3: The key features and rules of an N^2 matrix

The construction of an N^2 Matrix therefore requires the identification of the system functions or elements, which should be identified and recorded on the leading diagonal. The size of the matrix is equal to the number of system elements or functions. Interconnections between functions or system elements are captured in the off-diagonal cells. By convention, outputs are in rows and inputs are in columns. Inputs from outside the system, and outputs from the system are shown in-line with their respective source or destination function or element.

Once constructed, an N^2 Matrix provides considerable information about the system.

Figure 4 shows the features possible that include:

- Critical Element – where a single element receives inputs from a number of other elements and transmits to those same elements. Failure of this element could result in overall system failure. This can indicate where redundancy might be appropriate.
- Sphere of influence of critical elements show the reach of the critical element within the whole system.
- Tightly bound groups of elements that represent potential candidate sub-systems or if across two or more actual sub-system in an existing system areas of concern and redesign.
- Complex interactions/dependencies.
- Nodal points are natural breaks between groups of system elements. The actual interactions at the nodal points signify the magnitude of the interfacing.
- Feedback loops where two elements are tightly bound.
- Cascading flows between element.

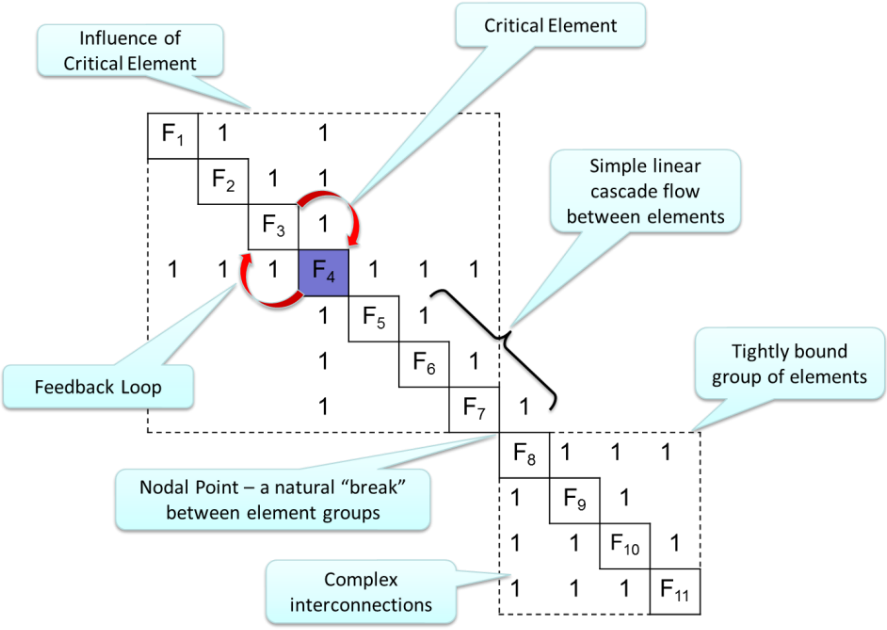

Figure 4: Possible N^2 Matrix Features

To Determine System Architectures (as part of concept design)

N^2 Analysis can help to uncover the natural structure of a system problem and thereby identify candidate architectures. It does demand the prior existence of logical functional model of the proposed system. There are several possible modelling tools that can fulfill this requirement, but for the purpose of this document, we will use a Function Flow Diagram [1].

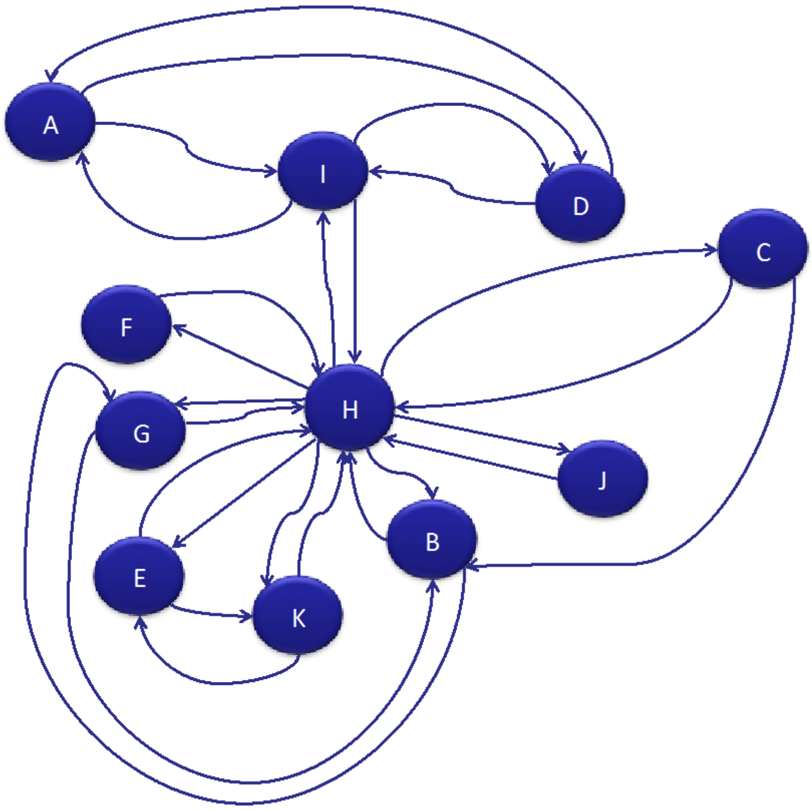

Figure 5: Example Function Flow Diagram – a logical Model of a Proposed System

If you are not familiar with this particular modelling tool the circles are functions of the system, the arrows are the inputs and outputs from the functions and also inputs and outputs external to the system[1].

Figure 5 can be used to construct the corresponding N^2 Matrix. The functions A to K are placed on the leading diagonal of the 11x11 matrix. The flows between functions are then captured in the remaining cells, with outputs in rows and inputs in columns. Figure 6 shows the corresponding N^2 Matrix for the system in Figure 5.

[1] There is a third element to Function Flow Diagram: the Terminator that is represented by a square. Terminators are elements external to the system of interest and are the sources of input flows and destination of output flows.

A			1					1		
	B					1	1			
	1	C					1			
1			D					1		
				E			1			1
					F		1			
	1					G	1			
	1	1		1	1	1	H	1	1	1
1			1				1	I		
							1		J	
				1			1			K

Figure 6: N^2 Matrix Corresponding to the system modelled in Figure 5

Figure 6, in its current form, is referred to as the "un-clustered matrix". The aim is to re-order the rows to expose the natural structure that exists in most systems. Indeed, Figure 7 shows this structure comprising a number of natural clusters of functions and a critical element centred on Function H.

A	1	1								
1	D	1								
1	1	I				1				
			C	1		1				
				B	1	1				
				1	G	1				
		1	1	1	1	H	1	1	1	1
						1	E	1		
						1	1	K		
						1			J	
						1				F

Figure 7: The re-ordered N^2 Matrix

Moving from the un-clustered Matrix to the clustered one can be accomplished by two means. Software packages do exist to perform the task, or it can be achieved by hand. At first glance the “by hand” method may appear somewhat daunting, but with a few simple “tricks” it can be made tractable.

Critical elements are usually easy to identify. Figure 7 clearly shows that Function H is a critical element, if we examine Figure 6 we can also see this because of the overly full column and row centred on H. This is easy to see in this example but in less clear examples, it can be deduced by calculating sum of every column and row. Figure 8 shows the outcome of this calculation for the system shown in Figure 6. We can clearly see the function H has a row sum of 8 and a column sum of 8, both being significantly larger than the sums of any other row or column – we have identified a potential critical element. In such cases, the critical element should be moved to the centre of the matrix and those linked elements placed sequentially adjacent to expose the cruciform of Figure 7.

Row Sums

A			1					1			2
	B					1	1				2
	1	C					1				2
1			D					1			2
				E			1			1	2
					F		1				1
	1					G	1				2
	1	1		1	1	1	H	1	1	1	8
1			1				1	I			3
							1		J		1
				1			1			K	2
2	3	1	2	2	1	2	8	3	1	2	

Column Sums

Figure 8: Un-clustered N^2 Matrix with row and column sums

To identify the clusters of functionality shown in Figure 7 a useful approach is to form the upper (or lower) triangular matrix by taking the interconnections of the “triangle” below the leading diagonal and placing them in their mirror positions in the upper “triangle”. To indicate the direction of the interconnection it is worthwhile changing the symbol. In Figure 9 the symbol “O” is used for the mirrored interconnections.

A			10					10		
	B	0				10	10			
		C					10			
			D					10		
				E			10			10
					F		10			
						G	10			
							H	10	10	10
								I		
									J	
										K

Figure 9: Upper Triangle form of the un-clustered N^2 Matrix

Figure 9 can now be used to identify potential clusters. For example Function A is tightly bound to Functions D and I. Function D is Tightly bound to Function I – hence there is a tightly bound triple between A, D and I as indicated by

Figure 10.

A			10					10		
	B	0				10	10			
		C					10			
			D					10		
				E			10			10
					F		10			
						G	10			
							H	10	10	10
								I		
									J	
										K

Figure 10: The tightly bound triple between Functions A, D and I

Function I also is tightly bound to the critical function H, but Functions A and D are not. Hence, the cluster can be exposed by ordering A, D then I – allowing Function I to be associated with the critical Function H.

A similar argument applies to Functions B, G and H as shown in Figure 11.

A			10					10		
	B	0				10	10			
		C					10			
			D					10		
				E			10			10
					F		10			
						G	10			
							H	10	10	10
								I		
									J	
										K

Figure 11: Tightly Bound Triple between B, G and H

Finally there is also binding between Functions E and K who also are associated with I as shown in Figure 12.

A			10					10		
	B	0				10	10			
		C					10			
			D					10		
				E			10			10
					F		10			
						G	10			
							H	10	10	10
								I		
									J	
										K

Figure 12: Tightly Bound Triple between B, G and H

These identified clusters and critical Element can now be used to reorder the matrix to yield Figure 7.

To Capture and Record Systems Interfaces for Management and Analysis

The N^2 Matrix provide a very simple yet powerful visual way of capturing and recording the interfaces in an system. This makes N^2 Analysis highly useful for the management of interfaces and also for the analysis of systems.

To create an N^2 Matrix for an existing system requires either the actual system or some suitable representation from which the interfaces can be identified. For complex systems this would be performed initially at a high level of generality in order to manage the complexity. Deciding on the level is a matter of problem context, but typically N^2 Matrices become difficult to comprehend and manage one the number of elements represented exceeds 15.

Figure 13 shows a typical domestic toaster that has a three slots, a knob to select the number of slots that heat-up, a knob for adjusting a timer that when complete cuts the electric power to the selected elements, and a lever to raise and lower the bread/toast.

Figure 13: A Typical Domestic Toaster.

Figure 14 shows a corresponding N^2 Matrix for this toaster to highlight the major interfaces. In constructing this matrix the “User” was included as a system element. This is a matter of choice and what the Analyst wishes to show or understand. Equally, the matrix does not show the internal detail, particularly the physical architectural elements. For instance on this particular toaster the casing supports and aligns the elements and there is a physical interface between these elements. In this example this information has been excluded to highlight the more “operational” type interfaces. An N^2 Matrix is a representation of reality – a model – and it is up to the “modeler” to decide the level of detail. It is also incumbent on the “modeler” to explain the assumptions made when constructed an N^2 Matrix.

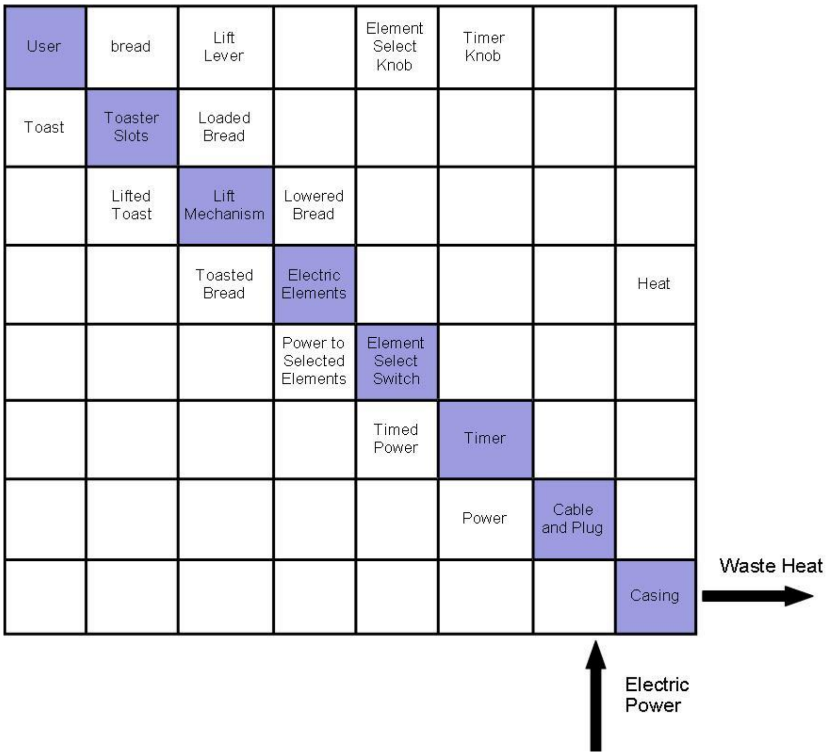

Figure 14: An N^2 Matrix for the Typical Domestic Toaster.

What Goes Wrong: The limitations of N^2 Analysis

N^2 Analysis is a very simple but powerful tool for capturing and exploring the interconnectivity in a system whether proposed or actual. Like all tools it is not without its drawbacks and the following gives some of the issues with its use are given below with advice on avoiding, and recovering from, the problem:

- When using N^2 Analysis to explore and identify candidate architectures, the analysis is only as good as the functional model. Whatever modelling tool is used, if it not complete or contains an error then this will be transferred to the N^2 Matrix. Functional models should always be independently reviewed to provide confidence of their accuracy.
- It is possible to "miss" certain structures hidden inside problem particularly with large N^2 matrices. The use of software often doesn't help and indeed reliance on software can result in less understanding of potential structures often found by manual exploration of the N^2 matrix.
- Using an N^2 matrix to record the presence and type of interface does require attention to detail. The obvious interfaces are often captured, but the less obvious ones such as thermal or noise interfaces are not.

Bibliography

References to N² Analysis appear in many texts, but unfortunately most are glib and ephemeral. There is at present no recommended seminal text on the tool.

History

The N^2 Matrix is attributed to Robert J. Lano, a Systems Engineering working for TRW in the 1970s and first published in a 1977 TRW internal report. Since then it has had many advocates particularly NASA and in recent years, under its pseudonym DSM, become popular in the Project Management community as a way of allocating tasks and resources.

References

[1] Burge S. "Flow Diagram" www.burgehugheswalsh.co.uk